A Hotelling-Downs Game for Strategic Candidacy with Binary Issues

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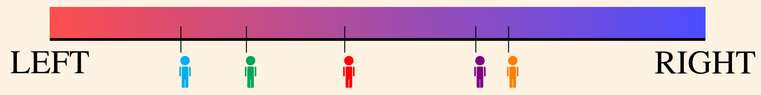
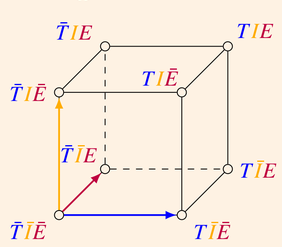
Motivation

- Candidates in elections can be strategic in order to win (or to have somebody else, *that they prefer*, win).
- A common model to capture this strategic behavior is the **Hotelling - Downs** model, where the *political spectrum* is represented by the real line.



- Seeing politics as a matter of *right or left* on a line seems quite reductive.
- We will model political opinions by positions *for* or *against* a certain predefined set of binary issues. e.g.
  - ▷ higher taxes (T)
  - ▷ immigration (I)
  - ▷ euthanasia (E)
  - ▷ raising the retirement age
  - ▷ ...



- Do candidates have an incentive to *deviate* from their truthful opinion? How will they act strategically in this setting? Do we have stable states?

Related Concepts in the Literature

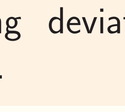
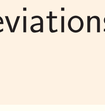
- Strategic candidacy
  - ▷ candidates can abstain, aiming to *get a better winner*.
- Hotelling-Downs model
  - ▷ strategic positioning of selfish players on a spatial dimension.
- Facility location problems
  - ▷ optimum location of a new facility w.r.t. a given metric space.
- Voronoi games
  - ▷ strategic positioning of players on a metric space, maximizing the amount of points closest to them.

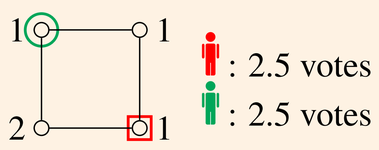
The Model

- **Hypercube** of *all possible opinions* over the binary issues.
- A voter's opinion is represented as a position in this hypercube (and so, each vertex has an associated *weight* of voters with such an opinion).
- Candidates will run for the election by *announcing* an opinion that they (supposedly) stand for.
- They may be willing to strategize (i.e. announce an *untruthful opinion*) to *get a better outcome* from the election.
  - ▷ Their *payoff* is determined by a fixed preference they have over the other candidates (in which they always strictly prefer themselves).

e.g.  : ≻ ∼ ≻ ≻ ...

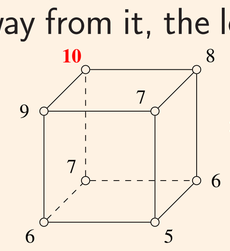
  - ▷ These preferences might even be narcissistic (indifferent among the rest).
- Winner of the election is decided by a variant of the plurality voting rule:
  - ▷ Each voter gets 1 vote, which is divided equally among the closest candidates.
  - ▷ deterministic tie-breaking rule:

 ▷  ▷ ...



Restrictions to the Setting

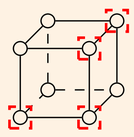
Single-peaked distribution of voters: There is a most popular opinion (peak) such that, the more we walk away from it, the less voters we find.



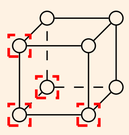
⇒ e.g. uniform distribution

Candidates' strategy sets: Candidates may only be willing to announce a subset of all possible positions. This set may be assumed:

- Connected



- A Ball around the *truthful* position.



Solution Concepts

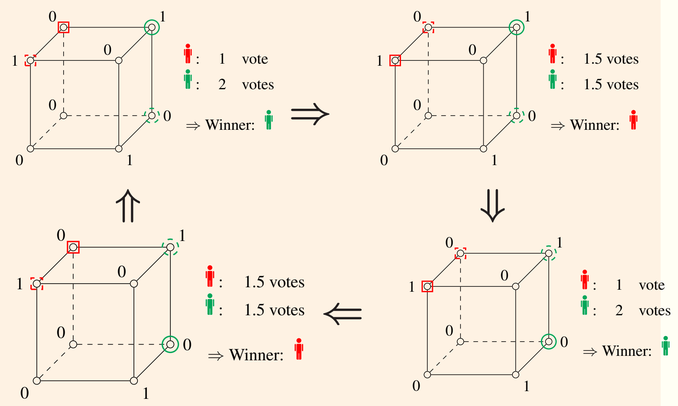
- Nash equilibrium (NE): stable state with respect to unilateral improving deviations from candidates.
- t-Local equilibrium (t-LocEq): stable state with respect to unilateral improving deviations from candidates to positions at distance $\leq t$ from the current one.

Existence of a Local Equilibrium: Negative Result

In general, no guarantee of existence of a 1-LocEq even with 2 candidates and 3 issues.

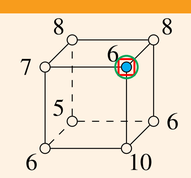
→ Deciding about the existence of a t-LocEq is NP-hard, for all $t \geq 2$.

What about $t = 1$? (open)



2 Candidates: Unrestricted Voter Distribution

- Majoritarian outcome: the position that takes the majoritarian value on each issue separately.

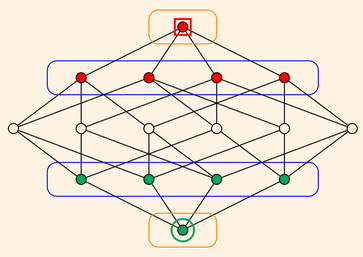


Guaranteed existence of 2-LocEq for 2 candidates, odd number of voters, and if they both can take the majoritarian outcome.

→ no such guarantee for a 3-LocEq (under the same conditions).

2 Candidates: Single-Peaked Voter Distribution

- Guaranteed existence of a NE with 2 candidates, under a **single-peaked distribution** of voters, when the candidate favored by the tie-breaking can *take* the peak position.
  - ▷ In particular, there always exists a NE with 2 candidates under a **uniform distribution**.



→ No guarantee for a 1-LocEq with 3 candidates (with *fixed* preferences) and 2 issues, even under a uniform distribution.

2 Candidates: 1-Local Equilibria and Candidates Strategies

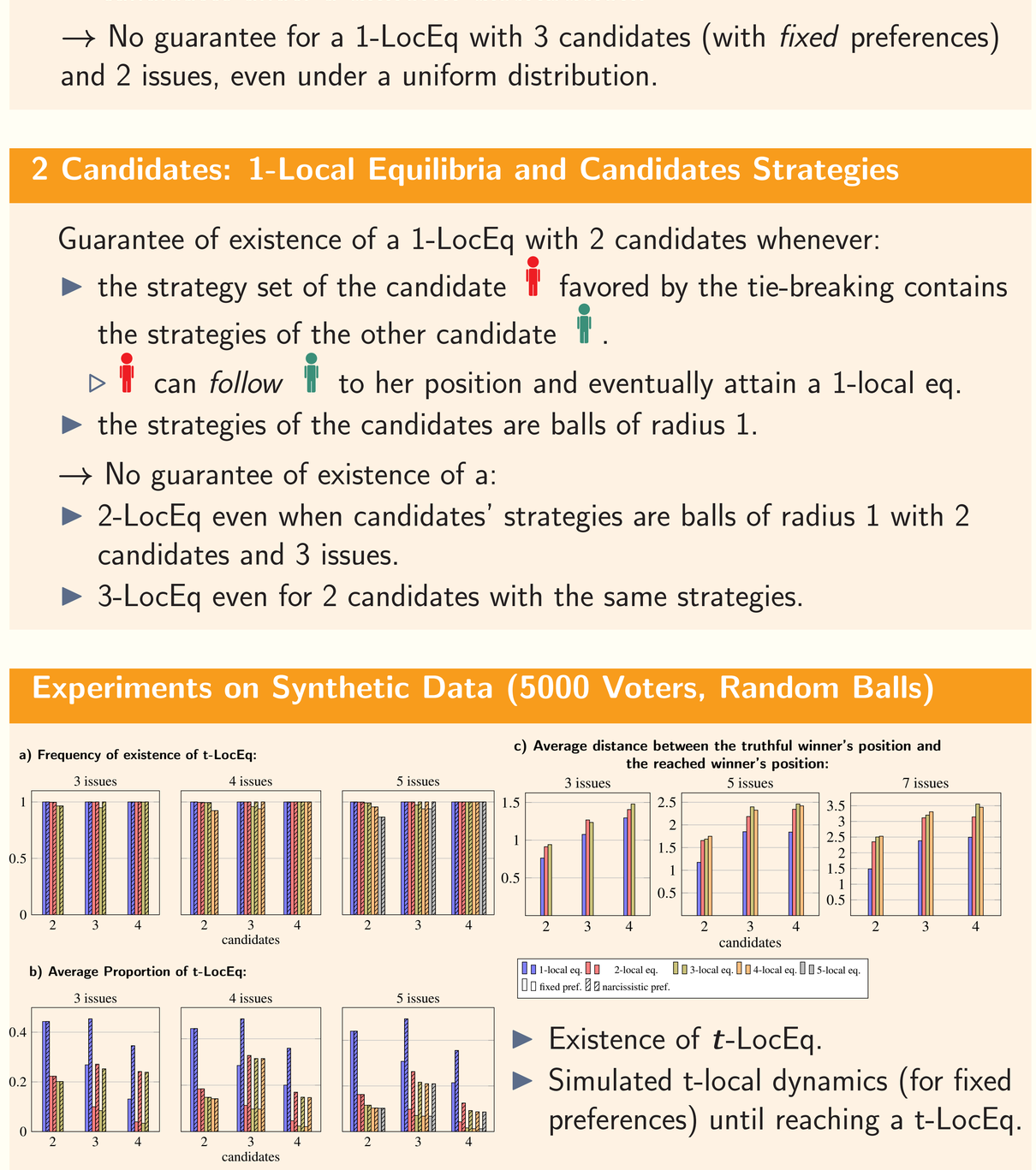
Guarantee of existence of a 1-LocEq with 2 candidates whenever:

- the strategy set of the candidate favored by the tie-breaking contains the strategies of the other candidate .
  - ▷ can *follow* to her position and eventually attain a 1-local eq.
- the strategies of the candidates are balls of radius 1.

→ No guarantee of existence of a:

- 2-LocEq even when candidates' strategies are balls of radius 1 with 2 candidates and 3 issues.
- 3-LocEq even for 2 candidates with the same strategies.

Experiments on Synthetic Data (5000 Voters, Random Balls)



- Existence of t-LocEq.
- Simulated t-local dynamics (for fixed preferences) until reaching a t-LocEq.

Perspectives and Future Work

- Other voting rules?
- Consideration of abstention?
- Correlation between positions?
- Strategic behavior from both voters and candidates?
- Increasing the score instead of a better winner?